On Parity-Violating Three-Nucleon Interactions and the Predictive Power of Few-Nucleon EFT at Very Low Energies

Harald W. Grießhammer[a,1] and **Matthias R. Schindler**[a]

[a] *Center for Nuclear Studies, Department of Physics,*
The George Washington University, Washington DC 20052, USA

Abstract

We address the typical strengths of hadronic parity-violating three-nucleon interactions in "pion-less" Effective Field Theory in the nucleon-deuteron (iso-doublet) system. By analysing the superficial degree of divergence of loop diagrams, we conclude that no such interactions are needed at leading order, $\mathcal{O}(\epsilon Q^{-1})$. The only two distinct parity-violating three-nucleon structures with one derivative mix $^2S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ waves with iso-spin transitions $\Delta I = 0$ or 1. Due to their structure, they cannot absorb any divergence ostensibly appearing at next-to-leading order, $\mathcal{O}(\epsilon Q^0)$. This observation is based on the approximate realisation of Wigner's combined $SU(4)$ spin-isospin symmetry in the two-nucleon system, even when effective-range corrections are included. Parity-violating three-nucleon interactions thus only appear beyond next-to-leading order. This guarantees renormalisability of the theory to that order without introducing new, unknown coupling constants and allows the direct extraction of parity-violating two-nucleon interactions from three-nucleon experiments.

---

[1]Corresponding author; email: hgrie@gwu.edu

arXiv:1007.0734v2 [nucl-th] 24 Jul 2010

1   Introduction

An international effort is under way to map out the weak portion of the nuclear force at low energies, stimulated in particular by the advancement of slow-neutron facilities such as the SNS (Oak Ridge), NIST (Gaithersburg), ILL (Grenoble), PSI (Villingen), FRM-II (München). Experiments on few-nucleon systems provide an excellent opportunity to study the weak interactions between free hadrons via parity-violating (PV) effects, see e.g. [1–3]. However, their interpretation requires adequate theoretical support: The consistency of different data sets must carefully be checked in one model-independent framework in order to account for the high complexity of such experiments which makes their systematic errors difficult to assess; binding effects must be taken into account with reliable error-estimates; and the PV interaction strengths must be extracted from PV observables using minimal theoretical bias, with the long-term goal of relating them to the parameters of the Standard Model, e.g. by lattice calculations [4].

These criteria are met by calculations in Effective Field Theories (EFTs). They describe few-nucleon systems with *a priori* estimates of theoretical uncertainties. Of particular interest to slow-neutron and other very-low energy facilities is the "pion-less" version EFT($\not\pi$) in which the dynamical degrees of freedom are only nucleons, see e.g. [5,6] for recent reviews. Its range of applicability is limited to momenta smaller than the pion mass m_π, i.e. energies of up to a dozen MeV. This allows for a systematic expansion of all observables in a generic low-momentum scale Q in units of this breakdown scale. For typical low momenta like the inverse scattering lengths of the two-nucleon bound-states, $\gamma \approx 45$ MeV, the expansion parameter is usually found to be $Q \approx \frac{1}{[3...5]}$. Since external momenta provide additional scales, the expansion parameter grows when these are significantly larger than γ and finally reaches unity for momenta of the order of the pion mass. In the parity-conserving (PC) sector, calculations are routinely performed at next-to-next-to-leading order N^{2}LO with typical accuracies of $\lesssim 4\%$. Another expansion parameter $\epsilon \approx 10^{-6}$ is provided by the PV strength relative to the PC one. EFT($\not\pi$) was first used in the PV sector in Ref. [7], with a comprehensive description given in Ref. [3]. In the two-nucleon system, only 5 independent PV parameters exist in the leading-order Lagrangean, $\mathcal{O}(\epsilon Q)$ [3, 8, 9]. They have to be determined from experiment. No further PV 2N interactions enter at NLO, i.e. when one power of Q is added. This allows one to compare data, subtract binding effects and extract PV interactions model-independently with $\lesssim 10\%$ accuracy in NN observables, matching the projected uncertainties of the most ambitious experiments.

However, the number of feasible low-energy experiments in the PV few-nucleon sector is limited. A complete data set to determine the PV parameters will most likely include observables with 3 and more nucleons, where e.g. neutron-neutron interactions are probed without the need for free neutron targets. There are also indications of better PV signals in light nuclei, like an increased neutron spin rotation in deuterium relative to hydrogen [10].

The extraction could be thwarted if parity-violating three-nucleon interactions (3NIs) contribute at the $\gtrsim 10\%$-level, i.e. at LO or NLO in EFT($\not\pi$). A PV 3NI will enter at some order as a manifestation of the interactions underlying EFT($\not\pi$). Since every PV 3NI requires one additional experiment to determine its strength, its appearance at LO or NLO

would exacerbate the problem of determining the PV 2NIs from few-nucleon data. In the strong sector of EFT($\not\pi$), a parity-conserving 3NI is expected only at N²LO, $\mathcal{O}(Q^0)$, but already enters at LO, $\mathcal{O}(Q^{-2})$, because the anomalously large NN scattering lengths force a non-trivial renormalisation. Only with a PC 3NI are observables insensitive to physics at short distance scales. The PC 3NI strength is determined from one three-nucleon datum, leading to the Phillips line [11] and Efimov effect [12]; see e.g. [5, 6] for reviews. Such a non-perturbative renormalisation may interfere with PV 3NIs and promote them to lower orders than simplistically expected.

We show that no PV 3NI enters at LO ($\mathcal{O}(\epsilon Q^{-1})$) or NLO ($\mathcal{O}(\epsilon Q^0)$) in the nucleon-deuteron system, the only 3N system which can be tested experimentally. We draw from Ref. [13, 14], where the general method to determine the divergence structure of 3N amplitudes was presented. Section 2 briefly recalls those aspects of PC and PV EFT($\not\pi$) needed here. We then proceed in two steps: Naïve dimensional analysis in Sec. 3.1 shows that there are no PV 3NIs at leading order. We then construct all PV 3NIs with only one derivative in Sec. 3.2. Section 3.3 discusses which PV nucleon-deuteron scattering contributions arise at NLO. In Sec. 3.4, we show that the constructed PV 3NIs do not match the structure of possible divergences, so that no PV 3NIs exist even at NLO. A summary with potential limitations and extensions of this work concludes the article.

2   Interactions and UV Limit

2.1   Parity-Conserving Part

In the parity-conserving three-nucleon sector of EFT($\not\pi$), we follow the conventions of Ref. [15]. The pertinent pieces of the parity-conserving Lagrangean up to NLO are:

$$
\mathcal{L}_{\mathrm{PC}} = N^\dagger(i\partial_0 + \frac{\vec{\partial}^2}{2M})N - y\left[d_t^{i\dagger}(N^T P_t^i N) + \mathrm{H.c.}\right] - y\left[d_s^{A\dagger}(N^T P_s^A N) + \mathrm{H.c.}\right] \tag{2.1}
$$
$$
+ d_t^{i\dagger}\left[\Delta_t - c_{0t}\left(i\partial_0 + \frac{\vec{\partial}^2}{4M} + \frac{\gamma_t^2}{M}\right)\right]d_t^i + d_s^{A\dagger}\left[\Delta_s - c_{0s}\left(i\partial_0 + \frac{\vec{\partial}^2}{4M} + \frac{\gamma_s^2}{M}\right)\right]d_s^A
$$
$$
+ \frac{y^2 M\, H_0(\Lambda)}{3\Lambda^2}\left[d_t^i(\sigma_i N) - d_s^A(\tau_A N)\right]^\dagger\left[d_t^i(\sigma_i N) - d_s^A(\tau_A N)\right] + \dots
$$

The nucleon field N has mass M. The spin-triplet and spin-singlet dibaryon fields d_t and d_s are introduced as auxiliary fields with the quantum numbers of the corresponding S-wave two-nucleon states to simplify calculations [16, 17]. With σ_i (τ_A) denoting Pauli matrices in spin (iso-spin) space, $P_t^i = \frac{1}{\sqrt{8}}\tau_2\sigma_2\sigma_i$ and $P_s^A = \frac{1}{\sqrt{8}}\tau_2\tau_A\sigma_2$ project the two-nucleon state onto the ³S₁ and ¹S₀ partial waves (in the notation $^{2S+1}l_J$, with S the spin, l the orbital angular momentum and J the total angular momentum). The parity-conserving $^2S_{\frac{1}{2}}$-wave three-nucleon interaction in the last line has strength $H_0(\Lambda)$ which depends on a regulator Λ, see below. We choose $y^2 = 4\pi/M \sim Q^0$. The LO parameters $\Delta_{s/t}$ are determined from low-energy data, e.g. the poles of the NN S-wave amplitudes at i$\gamma_{s/t}$, and are the only terms

of unnatural size, $\Delta_{s/t} \sim Q^{-1}$. The auxiliary-field propagators are at leading order:

$$D_{s/t}(q_0, \vec{q}) = \frac{1}{\gamma_{s/t} - \sqrt{\frac{\vec{q}^2}{4} - Mq_0 - \mathrm{i}\epsilon}} \ . \tag{2.2}$$

The parameters $c_{0s/t}$ enter at NLO, determined for example by the effective ranges [15, 18].

The Faddeev equations for the half-offshell amplitudes of nucleon-deuteron scattering at LO in the lth partial wave before wave-function renormalisation were first derived by Skorniakov and Ter-Martirosian [19]. Generalisation to full-offshell amplitudes is straightforward. The kinematics in the centre-of-mass system is specified in Fig. 1, with total non-relativistic energy E; momentum $\vec{k}$ for the incoming deuteron; momentum $\vec{p}$ for the outgoing one. The amplitude for half-offshell momenta $p = |\vec{p}|$ is found by setting the incoming leg on-shell, $E = \frac{3\vec{k}^2}{4M} - \frac{\gamma_t^2}{M}$; the on-shell point is in addition at $p = k$. The
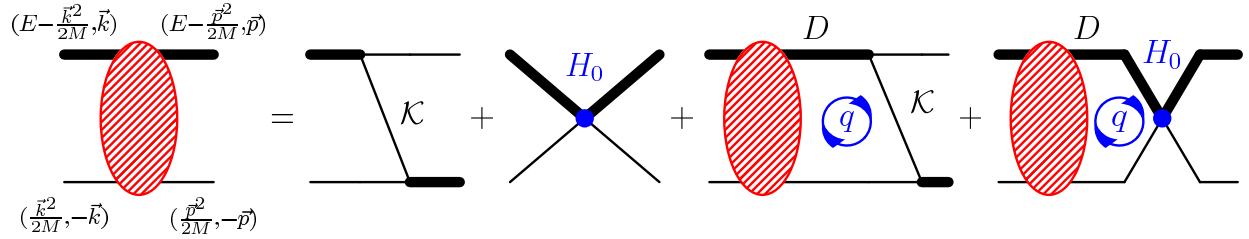

Figure 1: Three-nucleon scattering equation. Thick line: two-nucleon auxiliary-field propagator $D_{s/t}$ ($^1S_0/^3S_1$); thin line ($\mathcal{K}$): propagator of the exchanged nucleon; H_0: PC 3NI.

propagator of the exchanged nucleon, projected onto orbital angular momentum l, is

$$\mathcal{K}^{(l)}(E; q, p) := \frac{1}{2} \int\limits_{-1}^{1} \mathrm{d}\cos\theta \, \frac{P_l(\cos\theta)}{p^2 + q^2 - ME + pq\cos\theta} = \frac{(-1)^l}{pq} Q_l\left(\frac{p^2 + q^2 - ME}{pq}\right), \tag{2.3}$$

with $\theta = \angle(\vec{p}; \vec{q})$ and $P_l(z)$ ($Q_l(z)$) the lth Legendre polynomial of the first (second) kind with complex argument [20].

Two spin channels exist in the 3N system. The total spin $S = \frac{3}{2}$ (quartet) channel only receives contributions from combining the nucleon with the spin-triplet auxiliary field d_t, while both d_t and the spin-singlet auxiliary field d_s contribute in the $S = \frac{1}{2}$ (doublet) channel. With the two configurations d_tN and d_sN, it is convenient to follow Ref. [13, 14, App. A.1] in representing operators $\mathcal{O}$ by a 2×2-matrix in the so-called cluster-decomposition space:

$$\mathcal{O} = N_{b\beta}^\dagger \left(d_{t,j}^\dagger, \ d_{s,B}^\dagger \right) \begin{pmatrix} \mathcal{O}(Nd_t \to Nd_t)_i^j & \mathcal{O}(Nd_s \to Nd_t)_A^j \\ \mathcal{O}(Nd_t \to Nd_s)_i^B & \mathcal{O}(Nd_s \to Nd_s)_A^B \end{pmatrix}^{b\beta}_{a\alpha} \begin{pmatrix} d_t^i \\ d_s^A \end{pmatrix} N^{a\alpha} \ . \tag{2.4}$$

Operators thus act in the direct tensor-product space **spin** $\otimes$ **iso-spin** $\otimes$ **cluster** and carry the following indices: vector i, j, iso-vector A, B, spinor α, β and iso-spinor a, b. The latter

two will often be suppressed for convenience. The LO offshell amplitude $t_q^{(l)}(E; k, p) \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix} \right)$ in the spin-quartet channels is the solution to the integral equation

$$t_q^{(l)}(E; k, p) = -4\pi \, \mathcal{K}^{(l)}(E; k, p) + \frac{2}{\pi} \int_0^\Lambda dq \, q^2 \, \mathcal{K}^{(l)}(E; q, p) \, D_t(E - \frac{q^2}{2M}, q) \, t_q^{(l)}(E; k, q) \; . \quad (2.5)$$

The regulator Λ is used in the following to study the UV limit of Eq. (2.5). In cutoff regularisation, Λ is equal to or larger than the scale at which EFT($\not{\pi}$) breaks down.

For $S = \frac{1}{2}$, the amplitude $t_{d,XY}^{(l)}$ stands for the $Nd_X \to Nd_Y$-process, where $X, Y = s$ or t. For example, $t_{d,ts}^{(l)}$ stands for $Nd_t \to Nd_s$. Following e.g. Ref. [15, App. A], the full-offshell amplitude is the 2×2 matrix which solves

$$\begin{pmatrix} t_{d,tt}^{(l)} & t_{d,st}^{(l)} \\ t_{d,ts}^{(l)} & t_{d,ss}^{(l)} \end{pmatrix} (E; k, p) = 2\pi \left[\mathcal{K}^{(l)}(E; k, p) \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix} + \delta_0^l \frac{2H_0(\Lambda)}{\Lambda^2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right]$$
$$- \frac{1}{\pi} \int_0^\Lambda dq \, q^2 \left[\mathcal{K}^{(l)}(E; q, p) \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix} + \delta^{l0} \frac{2H_0(\Lambda)}{\Lambda^2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right] \qquad (2.6)$$
$$\times \begin{pmatrix} D_t & 0 \\ 0 & D_s \end{pmatrix} (E - \frac{q^2}{2M}, q) \begin{pmatrix} t_{d,tt}^{(l)} & t_{d,st}^{(l)} \\ t_{d,ts}^{(l)} & t_{d,ss}^{(l)} \end{pmatrix} (E; k, q) \; ,$$

and the half-offshell amplitude with an incoming nucleon and deuteron is obtained by multiplying with the column vector $\left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right)$ from the right and setting $E = \frac{3\vec{k}^2}{4M} - \frac{\gamma_t^2}{M}$.

According to a simplistic dimensional estimate, the three-nucleon interaction H_0 is of higher order and should thus not be included in Eq. (2.6). However, a detailed analysis of nd scattering in the ${}^2S_{\frac{1}{2}}$-wave [21] shows significant dependence on Λ in the solution of the three-body equation without H_0. This cutoff-dependence is removed by promoting the 3NI to leading order. Its strength H_0 is determined from one three-nucleon datum like the triton binding energy. The Phillips line [11] and Efimov effect [12] emerge as the physically observable remnants of the UV limit-cycle which describes its renormalisation-group running, see e.g. [5, 6] for reviews.

In order to investigate whether a similar promotion of higher-order terms occurs for PV 3NIs in EFT($\not{\pi}$), we must consider the UV-limit of the half-offshell momenta of (2.5/2.6). For $p, q \gg \sqrt{ME}, k, \gamma_{s/t}$, the auxiliary-field propagators are independent of external scales,

$$\lim_{q \gg \sqrt{ME}, \gamma_{s/t}} D_{s/t}(E - \frac{\vec{q}^2}{2M}, \vec{q}) = -\frac{2}{\sqrt{3}} \frac{1}{q} \; , \qquad (2.7)$$

as is the kernel $\mathcal{K}$. It has been demonstrated before [13, 14, 22] that the solutions to the resulting integral equations in the UV limit are linear combinations of

$$t_\lambda^{(l)}(q) := \lim_{q \gg \sqrt{ME}, k, \gamma_{s/t}} t_\lambda^{(l)}(E; k, q) \propto k^l \, q^{-s_l(\lambda)-1} \; , \qquad (2.8)$$

partial wave l	$s_l(\lambda = 1)$	$s_l(\lambda = -\frac{1}{2})$
0	$\pm 1.00624\ldots$ i	$2.16622\ldots$
1	$2.86380\ldots$	$1.77272\ldots$
$l \geq 2$	$\approx l + 1$	$\approx l + 1$

Table 1: Asymptotic coefficients $s_l(\lambda)$.

with the asymptotic exponents $s_l(\lambda)$ of the amplitudes at large half-offshell momenta $p, q \gg \sqrt{ME}, k, \gamma_{s/t}$ given for the lowest angular momenta in Table 1. For the spin-quartet channels, the spin-isospin parameter is $\lambda = -\frac{1}{2}$. For the spin-doublet channels, the situation is slightly more complicated. In the UV limit, the two auxiliary-field propagators (2.2) are identical and NN scattering becomes automatically Wigner-$SU(4)$-symmetric, i.e. symmetric under arbitrary combined rotations of spin and iso-spin [21, 23, 24]. The integral equations can then be decoupled by the transformation

$$
\begin{pmatrix} t_{\lambda=1}^{(l)} & t_{\lambda=-\frac{1}{2}\to\lambda'=1}^{(l)} \\ t_{\lambda=1\to\lambda'=-\frac{1}{2}}^{(l)} & t_{\lambda=-\frac{1}{2}}^{(l)} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} t_{d,tt}^{(l)} & t_{d,st}^{(l)} \\ t_{d,ts}^{(l)} & t_{d,ss}^{(l)} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad , \tag{2.9}
$$

leading to the Faddeev equation in the "Wigner-basis",

$$
\begin{pmatrix} t_1^{(l)} & t_{-\frac{1}{2}\to 1}^{(l)} \\ t_{1\to-\frac{1}{2}}^{(l)} & t_{-\frac{1}{2}}^{(l)} \end{pmatrix} (E; k, p) = 2\pi \left[\mathcal{K}^{(l)}(E; k, p) \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} + \delta_0^l \frac{2H_0(\Lambda)}{\Lambda^2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right]
$$

$$
- \frac{2}{\pi} \int_0^\Lambda \mathrm{d}q \, q^2 \left[\mathcal{K}^{(l)}(E; q, p) \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} + \delta^{l0} \frac{2H_0(\Lambda)}{\Lambda^2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right] \tag{2.10}
$$

$$
\times \begin{pmatrix} \Sigma & \Delta \\ \Delta & \Sigma \end{pmatrix} \left(E - \frac{q^2}{2M}, q \right) \begin{pmatrix} t_1^{(l)} & t_{-\frac{1}{2}\to 1}^{(l)} \\ t_{1\to-\frac{1}{2}}^{(l)} & t_{-\frac{1}{2}}^{(l)} \end{pmatrix} (E; k, q) \quad ,
$$

in which only the auxiliary-field propagators are not diagonal. While $\Sigma = \frac{1}{2}(D_t + D_s)$ is the "average" NN S-wave amplitude, $\Delta = \frac{1}{2}(D_t - D_s)$ parameterises the degree to which the $^3S_1$ and $^1S_0$-amplitudes differ [15, 25]. In the UV-limit, $\Delta = 0$ from (2.7), the components decouple in the Wigner-basis, and full-offshell amplitudes are diagonal. Off-diagonal elements are suppressed by $(\gamma_t - \gamma_s)/q$ in the UV limit and hence by one power of $(\gamma_s - \gamma_t)/m_\pi \sim Q$ [15, 25, 26]. The eigenvectors $t_1^{(l)}$ and $t_{-\frac{1}{2}}^{(l)}$ are also half-offshell amplitudes when $E = \frac{3\vec{k}^2}{4M} - \frac{\gamma_t^2}{M}$, see e.g. [15].

The amplitude $t_1^{(l)}$ is the eigenvector to the spin-isospin parameter $\lambda = 1$ and obeys the same integral equation as three spinless bosons; the amplitude $t_{-\frac{1}{2}}^{(l)}$ is the eigenvector to $\lambda = -\frac{1}{2}$ and shows the same asymptotics as the spin-quartet amplitudes Eq. (2.5). Since

5

the asymptotic coefficient $s_0(\lambda = 1) = \pm 1.0062 \ldots$ i in the ^{2}S$_{\frac{1}{2}}$-wave is imaginary, the two solutions are super-imposed [5, 6]:

$$t_1^{(l=0)}(q) \propto \frac{\cos[1.0062 \ldots \ln[q] + \delta]}{q} \ . \tag{2.11}$$

The relative phase δ is related to the PC 3NI strength $H_0(\Lambda)$ and thus determined by one 3N datum in the PC sector.

At NLO, exactly two corrections enter, as pictured in Fig. 2. Both are central, so that partial-waves do not mix. The first interaction is the insertion of one effective-range term
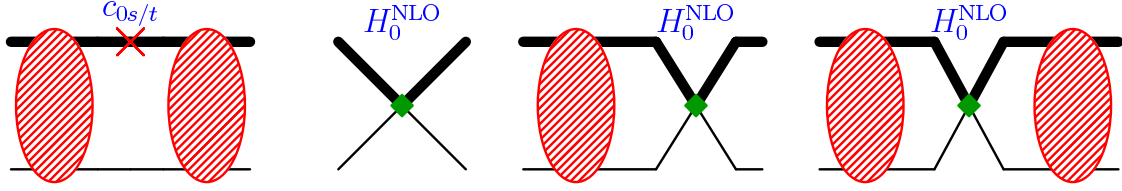

Figure 2: NLO corrections to the LO PC amplitudes: effective-range (cross); PC 3NI H_0^{NLO} (diamond). Crossed contributions not displayed.

$c_{0s/t}$ of the PC Lagrangean (2.1) in an auxiliary-field propagator. It is diagonal in the partial-wave basis since the two auxiliary fields d_s and d_t do not mix, cf. Eq. (2.1). In the Wigner-basis of Eq. (2.9), this translates to

$$\frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} c_{0t} & 0 \\ 0 & c_{0s} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} c_{0t} + c_{0s} & c_{0t} - c_{0s} \\ c_{0t} - c_{0s} & c_{0t} + c_{0s} \end{pmatrix} \ . \tag{2.12}$$

When $c_{0s/t} = \rho_{0s/t} M/2$ are determined by the effective ranges $\rho_{0s/t}$ [15, 18] with physical values $\rho_{0s} = 2.73 \, \text{fm}$ and $\rho_{0t} = 1.76 \, \text{fm}$, one finds that the off-diagonal elements are suppressed by a factor Q relative to the diagonal ones:

$$\frac{c_{0t} - c_{0s}}{c_{0t} + c_{0s}} \approx -0.22 \sim Q \ . \tag{2.13}$$

Off-diagonal elements again only contribute at higher orders in the power-counting:

$$\frac{1}{2} \begin{pmatrix} c_{0t} + c_{0s} & 0 \\ 0 & c_{0t} + c_{0s} \end{pmatrix} + \mathcal{O}(Q). \tag{2.14}$$

Determining $c_{0s/t}$ by different low-energy data leads to results which only differ by higher orders in Q. Off-diagonal elements are particularly strongly suppressed in Z-parameterisation [15, 18], where c_0 is determined by the residue of the pole in the NN amplitude as $c_{0s/t} = \frac{M\rho_{0s/t}}{2(1-\gamma_{s/t}\rho_{0s/t})}$, so that $\frac{c_{0t} - c_{0s}}{c_{0t} + c_{0s}} \approx \frac{1}{10} \sim Q^2$. The NN scattering amplitudes in the ^{1}S$_0$ and ^{3}S$_1$

channels therefore are to a good approximation Wigner-symmetric even when the effective-range corrections are taken into account.

The second NLO correction comes from including the NLO piece of the momentum-independent PC 3NI in the $^2S_{\frac{1}{2}}$-wave, $H_0^{\mathrm{NLO}}(\Lambda)$. It must be inserted once and re-adjusted to renormalise the NLO PC amplitudes [25, 26]. The NLO parity-conserving 3NI H_0^{NLO} of Eq. (2.1) in the Wigner-basis can be read off from Eq. (2.10) as proportional to $\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)$. Corrections are again suppressed by one power of Q and hence are of higher order [15, 25, 26].

That the LO 3N amplitude and its NLO corrections are diagonal in the Wigner-basis in the UV limit implies that in the parity-conserving sector, the spin-isospin parameter λ is approximately a good quantum number in the Wigner-SU(4) limit. This will be fundamental for showing in Sec. 3.4 that no PV 3NIs exist at NLO.

2.2 Parity-Violating Two-Nucleon Part

In contradistinction to the parity-conserving sector, parity-violating interactions may be included perturbatively at leading order since they are suppressed by $\epsilon \sim 10^{-6}$. The parity-violating two-nucleon Lagrangean at leading order, $\mathcal{O}(\epsilon Q)$, in the dibaryon formalism contains 5 coupling constants describing mixtures between S- and P-waves with the same total angular momentum and different iso-spin transitions, see Ref. [8, 9, 27] for details:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{PV}}^{\mathrm{LO}} = -\Big[& g^{(^3S_1-^1P_1)} d_t^{i\dagger} \left(N^T \sigma_2 \tau_2 \, \mathrm{i}\overset{\leftrightarrow}{\partial}_i N \right) \\
& + g_{(\Delta I=0)}^{(^1S_0-^3P_0)} d_s^{A\dagger} \left(N^T \sigma_2 \, \vec{\sigma} \cdot \tau_2 \tau_A \, \mathrm{i}\overset{\leftrightarrow}{\partial} N \right) \\
& + g_{(\Delta I=1)}^{(^1S_0-^3P_0)} \epsilon^{3AB} d_s^{A\dagger} \left(N^T \sigma_2 \, \vec{\sigma} \cdot \tau_2 \tau^B \overset{\leftrightarrow}{\partial} N \right) \\
& + g_{(\Delta I=2)}^{(^1S_0-^3P_0)} \mathcal{I}^{AB} d_s^{A\dagger} \left(N^T \sigma_2 \, \vec{\sigma} \cdot \tau_2 \tau^B \, \mathrm{i}\overset{\leftrightarrow}{\partial} N \right) \\
& + g^{(^3S_1-^3P_1)} \epsilon^{ijk} d_t^{i\dagger} \left(N^T \sigma_2 \sigma^k \tau_2 \tau_3 \overset{\leftrightarrow}{\partial}{}^j N \right) \Big] + \mathrm{H.c.}
\end{aligned}
$$
(2.15)

Here, $\overset{\leftrightarrow}{\partial} = \overset{\rightarrow}{\partial} - \overset{\leftarrow}{\partial}$, $\mathcal{I} = \mathrm{diag}(1, 1, -2)$ is a diagonal matrix in iso-vector space, and we have omitted couplings to external currents. The resulting leading-order PV two-nucleon scattering amplitude counts as $\mathcal{O}(\epsilon Q^0)$ because NN rescattering is enhanced by factors of $\Delta \sim Q^{-1}$ in the auxiliary-field formalism, while all other parameters are of natural size [15]. Any new interaction for S-P or S-F transitions must contain at least three derivatives and hence enters at N^{2}LO, $\mathcal{O}(\epsilon Q^3)$, which makes them relevant only for calculations with $\lesssim 3\%$ accuracy.

The three-nucleon amplitudes containing PV two-nucleon interactions are obtained from (2.15). But since the Effective Field Theory paradigm is that the Lagrangean contains *all* interactions allowed by symmetries, the question arises at which order the first PV 3NI appears. The lowest-order contributions will again connect S and P waves, analogous to the two-nucleon sector.

3 Parity-Violating Three-Nucleon Interactions in the Nucleon-Deuteron System

Simplistically, one may attempt to derive the order of the first PV 3NI by considering the tree-level diagrams in Fig. 3. The PV 2NI scales as ϵQ since it contains one derivative,
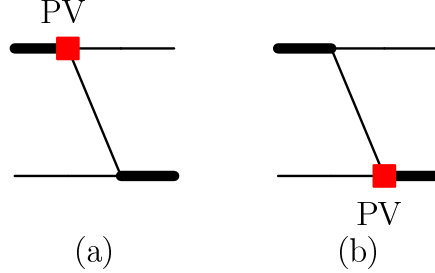

Figure 3: LO tree-level PV diagrams. Square (PV): the parity-violating 2N vertices $S \leftrightarrow P$.

see Lagrangean (2.15). As the nonrelativistic propagator of the exchanged nucleon (2.3) scales as Q^{-2}, the tree-level diagrams count as $\mathcal{O}(\epsilon Q^{-1})$. The first nonzero PV 3NI mixes S- and P-waves and thus contains one derivative, which translates into one power of Q. The PV 3NI therefore seems to enter at order ϵQ or N^{2}LO i.e. suppressed by 2 powers of Q against the first tree-level diagram involving PV 2NIs. In other channels, like P-D and S-F, a PV 3NI includes additional derivatives and hence additional powers of Q, suppressing their contribution even further. We therefore consider only PV 3NIs in the four channels of the Nd system which mix S- and P-waves and conserve total angular momentum:

$$^2S_{\frac{1}{2}} - {}^2P_{\frac{1}{2}} \; , \; ^2S_{\frac{1}{2}} - {}^4P_{\frac{1}{2}} \; ; \; ^4S_{\frac{3}{2}} - {}^2P_{\frac{3}{2}} \; , \; ^4S_{\frac{3}{2}} - {}^4P_{\frac{3}{2}} \tag{3.1}$$

As discussed above, large scattering lengths in the two-nucleon system lead to non-trivial renormalisation in the PC sector of both the two- and three-nucleon system, which in turn results in the promotion of, for example, the $^2S_{\frac{1}{2}}$ PC 3NI to lower order than the simplistic argument predicted. A more careful analysis in the PV sector is therefore warranted in particular in amplitudes which involve the critical $^2S_{\frac{1}{2}}$-wave in the initial or final state.

3.1 No PV Three-Nucleon Interaction at Leading Order

Consider the divergences generated by PV 2NIs in irreducible 3N diagrams. No loops and thus no divergences exist at tree level, Fig. 3. One-loop diagrams, Fig. 4, contain the LO PC 3N half-offshell amplitude $t_\lambda^{(l)}(E; k, q)$ once, with q the loop momentum. The LO PV 2N vertex itself contains a momentum-dependent piece proportional to loop and outgoing momenta. We symbolically denote the PV nucleon exchange as $(\vec{p}$ or $\vec{q}) \cdot \epsilon \vec{K}_{PV}$, where $\epsilon \vec{K}_{PV}$ is a stand-in for PV and PC couplings and spin-isospin-cluster structure, but does not contain the vectors $\vec{q}$ or $\vec{p} \sim \vec{k}$, or their magnitudes. Since it contributes one unit of angular momentum and violates parity, the transition amplitude relates states with orbital angular
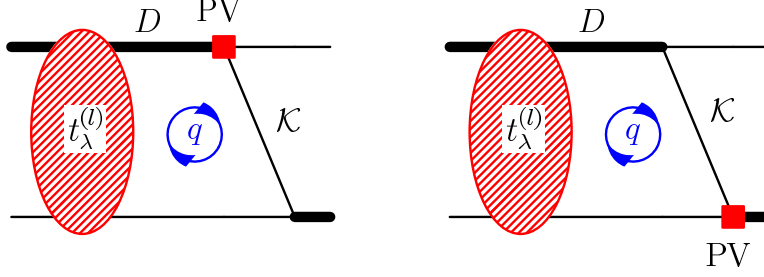
Figure 4: LO one-loop PV diagrams. Crossed contributions not displayed.

momenta l and $l \pm 1$. Performing the energy integration by picking up the spectator nucleon pole at $q_0 = \frac{q^2}{2M}$, the transition amplitude is made up of terms with the symbolic form

$$\int^{\Lambda} \mathrm{d}q \, q^2 \int \mathrm{d}\cos\theta \, \frac{(\vec{p} \text{ or } \vec{q}) \cdot \epsilon \vec{K}_{PV}}{p^2 + q^2 - ME + \vec{p} \cdot \vec{q}} \, P_l(\cos\theta) \, \frac{1}{\gamma_{s/t} - \sqrt{\frac{3q^2}{4} - ME}} \, t_\lambda^{(l)}(E; k, q) \quad (3.2)$$

for each component in cluster-space. As in (2.3), $\int \mathrm{d}\cos\theta$ projects the exchange-term to match the orbital angular momentum of the incoming PC amplitude $t_\lambda^{(l)}$.

The UV limit $q \gg \sqrt{ME}, p, k, \gamma_{s/t}$ of the amplitude is now constructed from that of its constituents: Either of the auxiliary-field propagators in the $^3S_1$ and $^1S_0$ state approaches $1/q$, see (2.7); the asymptotics of $t_\lambda^{(l)}$ is given by the spin-isospin-dependent exponent $s_l(\lambda)$ of (2.8) with Table 1; and only the intermediate-nucleon propagator and the PV vertex depend on θ, with the propagator expanded in powers of $\frac{p}{q}$. The asymptotics is therefore

$$\int^{\Lambda} \frac{\mathrm{d}q}{q^{2+s_l(\lambda)}} \int \mathrm{d}\cos\theta \, P_l(\cos\theta) \, (\vec{p} \text{ or } \vec{q}) \cdot \epsilon \vec{K}_{PV} \left[1 - \frac{\vec{p} \cdot \vec{q} - \frac{2}{\sqrt{3}} q \gamma_{s/t}}{q^2} + \mathcal{O}(q^{-2}) \right] \quad , \quad (3.3)$$

where $\mathcal{O}(q^{-2})$ denotes further corrections to the auxiliary-field and intermediate-nucleon propagators suppressed by two powers of p, k, $\gamma_{s/t}$ over q.

Combining the first term in brackets with the most divergent piece $\vec{q} \cdot \vec{K}_{PV}$ of the interaction, the most UV-dependent contribution for a given orbital angular momentum l of the PC amplitude thus seems to scale as $q^{-s_l(\lambda)}$, i.e. its superficial degree of divergence seems to be $\Delta(l; \lambda) = -\mathrm{Re}[s_l(\lambda)]$. Inspecting Table 1, one finds the diagram converges for nearly all partial waves, except when the PC amplitude is the $\lambda = 1$ part of the $^2S_{\frac{1}{2}}$-wave ($\lambda = 1$ and $l = 0$). Since $s_0(\lambda = 1) = 1.006 \ldots i$ is imaginary, the PV amplitude appears logarithmically divergent, with a phase determined by a PC three-nucleon datum, see Eq. (2.11). However, this amplitude is actually identically zero upon angular integration: $\int \mathrm{d}\cos\theta \, P_0(\cos\theta)\vec{q} = 0$. The next term in the bracket, $\vec{p} \cdot \vec{q}/q^2$, is nonzero after angular integration, $\int \mathrm{d}\cos\theta \, P_0(\cos\theta)\vec{q}(\vec{p} \cdot \vec{q})/q^2 \propto \vec{p}$, but suppressed by one more inverse power of q. This part converges thus with degree $\Delta = -1 - \mathrm{Re}[s_0(\lambda)] < 0$. For the other part of the

interaction, $\vec{p} \cdot \vec{K}_{PV}$, the angular integration from the "1"-term of the bracket is nonzero. The degree of divergence is again given by $\Delta = -1 - \mathrm{Re}[s_0(\lambda)] < 0$, and the PV S-to-P wave amplitude therefore scales as:

$$\lim_{q \to \infty} \mathcal{A}_{LO,\text{1-loop}}^{(\text{S}\to\text{P})} \sim (\vec{p} \cdot \vec{K}_{PV}) \, q^{-1-s_0(\lambda)} \to 0 \ , \ \ \Delta_{LO,\text{1-loop}}^{(\text{S}\to\text{P})}(\lambda) = -1 - \mathrm{Re}[s_0(\lambda)] \tag{3.4}$$

Further terms in the expansion are suppressed by more negative powers of q. From Table 1, one reads off $s_0(\lambda = 1) = 1.006\ldots\mathrm{i}$, $s_0(\lambda = -\frac{1}{2}) = 2.166\ldots$ and concludes that no divergences occur when LO PV 2NIs are convoluted with LO PC one-loop S-wave amplitudes, since the partial-wave-dependent degrees of divergence are:

$$\Delta_{LO,\text{1-loop}}^{(\text{S}\to\text{P})}(\lambda = 1) = -1 \ , \ \ \Delta_{LO,\text{1-loop}}^{(\text{S}\to\text{P})}(\lambda = -\frac{1}{2}) = -3.16\ldots \tag{3.5}$$

When convoluting with a PC P-wave amplitude, the angular integral involving the "1"-term is $\int \mathrm{d}\cos\theta \, P_1(\cos\theta)\vec{q} \neq 0$. The amplitude thus scales as:

$$\lim_{q \to \infty} \mathcal{A}_{LO,\text{1-loop}}^{(\text{P}\to\text{S})} \sim (\vec{p} \cdot \vec{K}_{PV}) \, q^{-s_1(\lambda)} \to 0 \ , \ \ \Delta_{LO,\text{1-loop}}^{(\text{P}\to\text{S})}(\lambda) = -\mathrm{Re}[s_1(\lambda)] \tag{3.6}$$

and, using Table 1, the superficial degrees of divergence are hence:

$$\Delta_{LO,\text{1-loop}}^{(\text{P}\to\text{S})}(\lambda = 1) = -2.86\ldots \ , \ \ \Delta_{LO,\text{1-loop}}^{(\text{P}\to\text{S})}(\lambda = -\frac{1}{2}) = -1.77\ldots \tag{3.7}$$

i.e. no divergence occurs. By time-reversal symmetry, LO one-loop contributions with the PC rescattering amplitude on the outgoing leg share the same divergence structure.

Consider now two-loop amplitudes, Fig. 5. The same argument applies separately for each integration when the loop momenta are $q \gg p$ or $p \gg q$. The overlapping divergence
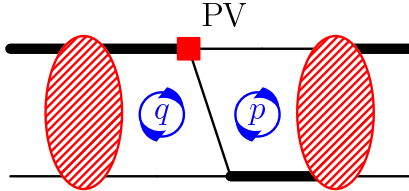

Figure 5: LO two-loop PV diagram. Crossed contributions not displayed.

for $p \sim q \gg \sqrt{ME}, k, \gamma_{s/t}$ leads to overall scaling

$$\lim_{q \to \infty} \mathcal{A}_{LO,\text{2-loop}}^{(\text{S}\leftrightarrow\text{P})} \sim (\vec{k} \cdot \vec{K}_{PV}) \, q^{1-s_0(\lambda_\text{S})-s_1(\lambda_\text{P})} \to 0 \ , \ \ \Delta_{LO,\text{2-loop}}^{(\text{S}\leftrightarrow\text{P})}(\lambda_\text{S}, \lambda_\text{P}) = 1 - \mathrm{Re}[s_0(\lambda_\text{S})+s_1(\lambda_\text{P})]$$
$$\tag{3.8}$$

where λ_S (λ_P) is the spin-isospin index of the S-wave (P-wave) PC 3N half-offshell amplitude. The superficial degrees of divergence are independent of the order of S- and P-waves:

$$\Delta_{LO,\text{2-loop}}^{(\text{S}\leftrightarrow\text{P})}(\lambda_\text{S} = \lambda_\text{P} = 1) = -1.86\ldots \ , \ \ \Delta_{LO,\text{2-loop}}^{(\text{S}\leftrightarrow\text{P})}(\lambda_\text{S} = 1, \lambda_\text{P} = -\frac{1}{2}) = -0.77\ldots \tag{3.9}$$

$$\Delta_{LO,\text{2-loop}}^{(\text{S}\leftrightarrow\text{P})}(\lambda_\text{S} = -\frac{1}{2}, \lambda_\text{P} = 1) = -3.03\ldots \ ; \ \ \Delta_{LO,\text{2-loop}}^{(\text{S}\leftrightarrow\text{P})}(\lambda_\text{S} = \lambda_\text{P} = -\frac{1}{2}) = -2.94\ldots \tag{3.10}$$

Since all amplitudes converge at LO, no PV 3NIs are needed for renormalisability.

10

3.2 Structure of PV Three-Nucleon Interactions

At LO, we demonstrated that no PV 3NI enters by explicitly showing that no divergence occurs in LO diagrams with PV 2NIs. The additional suppression by Q at NLO may come from one more power of loop momentum q, and hence may add one unit to the degrees of divergence in Eqs. (3.5/3.7/3.9). However, the number and complexity of possibly divergent diagrams is significantly larger and arguments based on the superficial degree of divergence become more elaborate, especially for overlapping divergences. In addition, recall that the superficial degree of divergence of any diagram provides only an upper bound. The actual degree of divergence can be lower when spin-isospin symmetry and the details of the interactions are taken into account. As this will be the case, we choose a different method.

In order to demonstrate that there are no PV 3NIs at NLO in the Nd system, we proceed in three steps: First, construct the spin-isospin-cluster structure of all PV 3NIs with only one derivative for S-P wave transitions; second, identify the corresponding structures in all NLO diagrams with PV 2NIs; and third, demonstrate that none of these matches the spin-isospin-cluster structure of the PV 3NIs, so that a promotion of PV 3NIs from the simplistic estimate N^{2}LO to NLO is not required as it cannot renormalise any potential divergence.

With the iso-doublet nucleon-deuteron system as both in and out state, a PV 3NI has $\Delta I \in \{0; 1\}$. A PV interaction with $\Delta I = 2$ requires an $I = 3/2$ state. Since the strong interactions considered here do not change isospin, returning to the $I = 1/2$ nucleon-deuteron system would not be possible without a second, highly suppressed insertion of another PV interaction. We therefore neglect the $\Delta I = 2$ PV 3NI. If an un-physical $I = 3/2$ state is found in both the initial and final state, a $\Delta I = 3$ PV 3NI would have to be considered as well. Since the transition amplitude relates S- and P-waves, the PV 3NI is expected to contain an odd number of derivatives. In addition, from Eqs. (3.4/3.6/3.8), the potentially divergent amplitudes are proportional to one power of a low-energy momentum $p \sim k$. Therefore, a PV 3NI must contain exactly one derivative. Further building blocks are the nucleon fields N, the spin and iso-spin Pauli matrices and the Levi-Cività symbols ϵ^{ijk} and ϵ^{ABC}. Indices must be saturated completely, and only spin and vector indices can relate. Unitarity, time-reversal symmetry and total angular momentum conservation must be taken into account, too.

Using *Mathematica* for algebraïc manipulations, we find that there is only one unique iso-scalar structure, with different variants related by Fierz transformations:

$$\left(N^\dagger N\right)\left(N^\dagger N\right)\left(N^\dagger \sigma^i \, \mathrm{i}\partial_i N\right) = \left(N^\dagger \sigma^i N\right)\left(N^\dagger N\right)\left(N^\dagger \, \mathrm{i}\partial_i N\right) \tag{3.11}$$

$$= \frac{1}{3}\left(N^\dagger \tau_A N\right)\left(N^\dagger \tau^A N\right)\left(N^\dagger \sigma^i \, \mathrm{i}\partial_i N\right) = -\frac{1}{3}\left(N^\dagger \sigma_j \tau_A N\right)\left(N^\dagger \sigma^j \tau^A N\right)\left(N^\dagger \sigma^i \, \mathrm{i}\partial_i N\right)$$

$$= -\frac{1}{5}\left(N^\dagger \sigma_j N\right)\left(N^\dagger \sigma^j N\right)\left(N^\dagger \sigma^i \, \mathrm{i}\partial_i N\right) = \text{ etc.}$$

In order to construct this interaction in the cluster-decomposition basis, Eq. (2.4), note that the components of the $^2\mathrm{S}_{\frac{1}{2}}$ state couple the auxiliary fields with a nucleon via two forms related by Fierz identities:

$$\sigma_i d_t^i N^b = -\left(\tau_A\right)^b{}_c \, d_s^A N^c \quad, \tag{3.12}$$

where the iso-spinor indices are made explicit. The corresponding $^2P_{\frac{1}{2}}$-wave components contain a spatial derivative whose index is saturated with another Pauli matrix:

$$d_t^i(\vec{\sigma}\cdot\overleftrightarrow{\partial})\sigma_i N^b = -d_s^A(\vec{\sigma}\cdot\overleftrightarrow{\partial})\left(\tau_A\right)^b{}_c N^c \tag{3.13}$$

For the $^2S_{\frac{1}{2}}$-$^2P_{\frac{1}{2}}$ $\Delta I = 0$ 3NI, spinor indices are contracted to form an iso-scalar:

$$\left[N^\dagger d_t^{j\dagger}\,\sigma_j\right]_a \delta_b^a\left[d_t^k\left(\sigma^i\,\mathrm{i}\overleftrightarrow{\partial}_i\,\right)\sigma_k\,N\right]^b + \text{H.c.} \tag{3.14}$$

$$= -\left[N^\dagger d_s^{A\dagger}\,\tau_A\right]_a \delta_b^a\left[d_t^k\left(\sigma^i\,\mathrm{i}\overleftrightarrow{\partial}_i\,\right)\sigma_k\,N\right]^b + \text{H.c.}$$

$$= \left[N^\dagger d_s^{A\dagger}\,\tau_A\right]_a \delta_b^a\left[d_s^B\left(\sigma^i\,\mathrm{i}\overleftrightarrow{\partial}_i\,\right)\tau_B\,N\right]^b + \text{H.c.}$$

$$= -\frac{9}{2}\left(N^\dagger N\right)\left(N^\dagger N\right)\left(N^\dagger \sigma^i\,\mathrm{i}\partial_i N\right)$$

The last line is the result of yet another Fierz transformation and establishes the identity between the only possible $\Delta I = 0$ PV 3NI with only one derivative and the cluster-decomposed form of the PV 3NI in the $^2S_{\frac{1}{2}}$-$^2P_{\frac{1}{2}}$ channel. That a PV 3NI with only one derivative can be re-written in this form is not surprising. With no derivative acting on two combinations containing N and $N^\dagger$ in (3.11), these two can be re-arranged in a relative S-wave and hence represented by the S-wave auxiliary fields. The derivative puts the remaining nucleon in a P-wave relative to the other two particles.

The $^2S_{\frac{1}{2}}$-$^2P_{\frac{1}{2}}$ $\Delta I = 1$ PV 3NI is obtained by inserting τ_3 as the $\Delta I = 1$ operator:

$$\left[N^\dagger d_t^{j\dagger}\sigma_j\right]_a (\tau_3)^a{}_b\left[d_t^i(\vec{\sigma}\cdot\mathrm{i}\overleftrightarrow{\partial})\sigma_i N\right]^b + \text{H.c.}$$

$$= -\left[N^\dagger d_s^{A\dagger}\tau_A\right]_a (\tau_3)^a{}_b\left[d_t^i(\vec{\sigma}\cdot\mathrm{i}\overleftrightarrow{\partial})\sigma_i N\right]^b + \text{H.c.}$$

$$= \left[N^\dagger d_s^{A\dagger}\tau_A\right]_a (\tau_3)^a{}_b\left[d_s^B(\vec{\sigma}\cdot\mathrm{i}\overleftrightarrow{\partial})\tau_B N\right]^b + \text{H.c.}$$

$$= \frac{9}{2}\left(N^\dagger N\right)\left(N^\dagger N\right)\left(N^\dagger\tau_3\sigma^i\,\mathrm{i}\partial_i N\right) = \text{ etc.} \tag{3.15}$$

As in the iso-singlet case, one can show with the help of algebraïc manipulation software that every PV 3NI with the above-mentioned building blocks, supplemented by τ_3, is proportional to this structure. Note that there is no parity-violating $^2S_{\frac{1}{2}}$-$^4P_{\frac{1}{2}}$ three-nucleon *contact* term. The Nd system can of course be in a $^4P_{\frac{1}{2}}$ channel.

The Lagrangean describing these interactions is therefore

$$\mathcal{L}_{\text{PV}}^{\text{3NI}} = \frac{y^2 M}{3\Lambda^3}\left[H_{\text{PV}}^{(\Delta I=0)}(\Lambda)\left[N^\dagger d_t^{j\dagger}\,\sigma_j - N^\dagger d_s^{A\dagger}\,\tau_A\right]\left[d_t^i(\vec{\sigma}\cdot\mathrm{i}\overleftrightarrow{\partial})\sigma_i N - d_s^B(\vec{\sigma}\cdot\mathrm{i}\overleftrightarrow{\partial})\tau_B N\right] \tag{3.16}$$

$$+ H_{\text{PV}}^{(\Delta I=1)}(\Lambda)\left[N^\dagger d_t^{j\dagger}\,\sigma_j - N^\dagger d_s^{A\dagger}\,\tau_A\right]\tau^3\left[d_t^i(\vec{\sigma}\cdot\mathrm{i}\overleftrightarrow{\partial})\sigma_i N - d_s^B(\vec{\sigma}\cdot\mathrm{i}\overleftrightarrow{\partial})\tau_B N\right]\right] + \text{H.c.} \,.$$
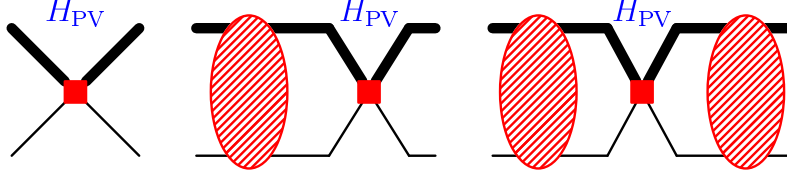
12

Figure 6: PV 3NI graphs which may absorb infinities. Crossed contributions not displayed.

The pre-factor was chosen in analogy to that of the PC 3NI in Eq. (2.1) such that the PV 3NIs $H_{\text{PV}}^{(\Delta I=0,1)}(\Lambda)$ have mass-dimension zero. If PV 3NIs are needed at NLO to cure divergences, H_{PV} may diverge for large Λ and enter through the diagrams in Fig. 6.

Consider now the spin-isospin-cluster structure of the PV 3NIs. The projector in cluster-configuration space onto the doublet-S wave was already constructed in Ref. [13, 14] and is supplemented here by that onto the $^2\text{P}_{\frac{1}{2}}$ channel. Both can also be read off from (3.12/3.13):

$$\mathcal{P}[^2\text{S}_{\frac{1}{2}}]_{iA} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sigma_i & 0 \\ 0 & \tau_A \end{pmatrix} \quad , \quad \mathcal{P}[^2\text{P}_{\frac{1}{2}}]_{iA} = (\vec{\sigma} \cdot \vec{e}) \frac{1}{\sqrt{3}} \begin{pmatrix} \sigma_i & 0 \\ 0 & \tau_A \end{pmatrix} \quad , \tag{3.17}$$

where $\vec{e}$ is the unit vector in the direction of the momentum of the auxiliary field $d_{s/t}$ in the centre-of-mass system. The vector index i and iso-vector index A are contracted with the respective indices of the auxiliary fields. Since this leaves only free spinor and iso-spinor indices, the resulting state

$$\mathcal{P}[^2l_{\frac{1}{2}}]_{iA} \begin{pmatrix} d_t^i \\ d_s^A \end{pmatrix} N \tag{3.18}$$

carries total spin $\frac{1}{2}$, and zero or one unit of orbital angular momentum l. The projectors are ortho-normalised as

$$\frac{1}{4\pi} \int d\Omega_e \, \mathcal{P}[^2l_{\frac{1}{2}}]_{iA} \, \mathcal{P}^\dagger[^2l'_{\frac{1}{2}}]^{iA} = \delta_{l'}^l \quad , \tag{3.19}$$

where the integration is over the direction of $\vec{e}$. A complete list of all S- and P-wave projectors of the three-nucleon system is given in an upcoming publication [28].

Projecting the PV 3NIs Eq. (3.16) onto an incoming $^2\text{S}_{\frac{1}{2}}$- and outgoing $^2\text{P}_{\frac{1}{2}}$-wave (or vice versa), one finds

$$i\mathcal{M}\left[^2\text{S}_{\frac{1}{2}} \to {}^2\text{P}_{\frac{1}{2}}, p, q\right]_{3\text{NI}} = A_{3\text{NI}} \left(H_{\text{PV}}^{(\Delta I=0)} + \tau^3 H_{\text{PV}}^{(\Delta I=1)}\right) \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \tag{3.20}$$

where the overall factor $A_{3\text{NI}}$ is a function of $E, k, p, q, \gamma_{s/t}$ and a scalar in spin-isospin-cluster space. Its exact form is not needed in the following. As pointed out in Sect. 2.1, a more convenient cluster-space basis in the UV limit is that in which the PC 3N amplitudes are

13

diagonal with spin-isospin parameters $\lambda = 1$ or $\lambda = -\frac{1}{2}$. The basis-change (2.9)

$$
\begin{aligned}
\mathrm{i}\mathcal{M}\left[^2\mathrm{S}_{\frac{1}{2}} \to {}^2\mathrm{P}_{\frac{1}{2}}, p, q\right]_{\mathrm{3NI}}^{\mathrm{Wigner}} &= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathrm{i}\mathcal{M}\left[^2\mathrm{S}_{\frac{1}{2}} \to {}^2\mathrm{P}_{\frac{1}{2}}, p, q\right]_{\mathrm{3NI}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \\
&= A_{\mathrm{3NI}}\left(H_{\mathrm{PV}}^{(\Delta I=0)} + \tau^3\, H_{\mathrm{PV}}^{(\Delta I=1)}\right)\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \quad\quad (3.21)
\end{aligned}
$$

shows that both PV 3NIs connect only the $^2\mathrm{S}_{\frac{1}{2}}(\lambda = 1)$-$^2\mathrm{P}_{\frac{1}{2}}(\lambda = 1)$ components, just as the PC 3NI. As shown in Sec. 2.1, the LO parity-conserving amplitudes are diagonal in Wigner space. Therefore, all diagrams of Fig. 6 have the same structure, i.e. they all connect only the $^2\mathrm{S}_{\frac{1}{2}}(\lambda = 1)$-$^2\mathrm{P}_{\frac{1}{2}}(\lambda = 1)$ components.

We re-emphasise the most important aspect of this construction: There are two and only two independent structures with $\Delta I \in \{0; 1\}$ available in the nucleon-deuteron system which contain exactly one derivative, and both are nonzero *only* in the $^2\mathrm{S}_{\frac{1}{2}}(\lambda = 1)$-$^2\mathrm{P}_{\frac{1}{2}}(\lambda = 1)$ channel, i.e. only the upper-left entry in the cluster matrix in the Wigner-basis is non-zero, see Eq. (3.21). This implies that any superficial divergence which does *not* share these quantum numbers cannot be cured by this PV 3NI. In particular, no divergences can appear at this order in the $^2\mathrm{S}_{\frac{1}{2}}$-$^4\mathrm{P}_{\frac{1}{2}}$ amplitudes or any other channel.

3.3 Contributions with PV 2NIs at Next-To-Leading Order

In the second step, we write down all NLO diagrams with PV 2NIs. PV 2NIs appear only at odd powers of Q. Thus, higher-order PV corrections only start contributing at order ϵQ, i.e. N²LO, where simplistic arguments suggest that PV 3NIs enter as well. The NLO contributions to PV nucleon-deuteron scattering therefore stem from the PC corrections presented in Sec. 2.1. The first set, namely effective-range corrections to Figs. 4 and 5, are shown in Fig. 7. The only other PC NLO correction, H_0^{NLO}, leads to the diagrams of Fig. 8.
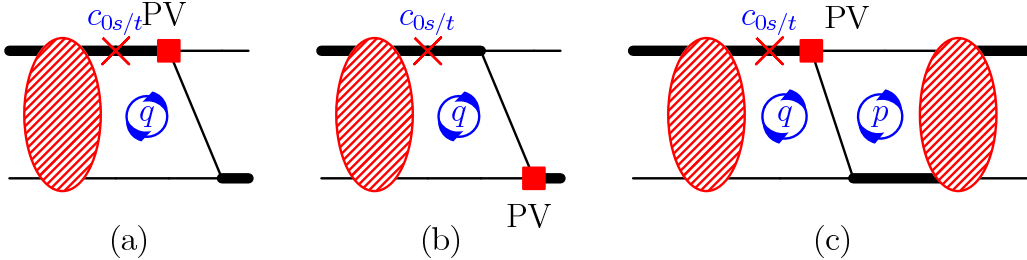

Figure 7: NLO PV diagrams, derived from Figs. 4 and 5 by adding one insertion of the effective-range correction to the deuteron propagator. Crossed contributions not displayed.

Since this 3NI appears only in the $^2\mathrm{S}_{\frac{1}{2}}$-wave, the LO amplitude $t_\lambda^{(l=0)}$ can be attached only on the side of H_0^{NLO}. Finally, a LO amplitude $t_\lambda^{(l)}$ can be inserted between the PC and PV interactions, see Fig. 9. These nucleon-deuteron rescattering contributions are found by convoluting NLO parity-conserving amplitudes with the LO PV kernels of Fig. 3.
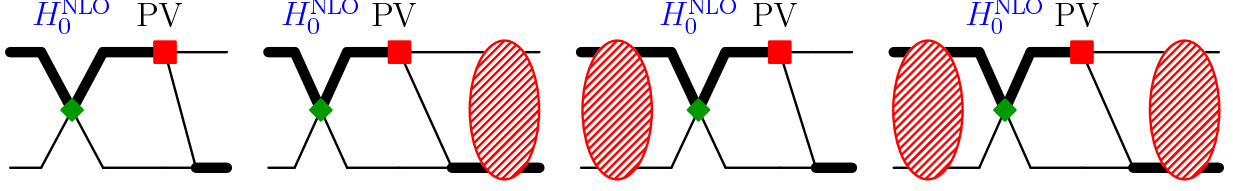
Figure 8: NLO PV diagrams with one insertion of the NLO PC 3NI correction. Crossed contributions and those with the PV interaction instead on the bottom right not displayed.
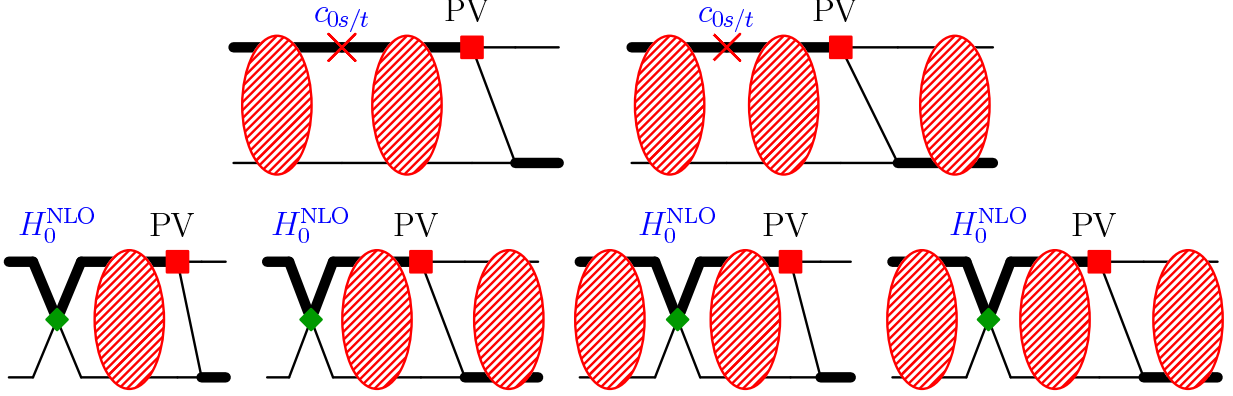

Figure 9: NLO PV diagrams with nucleon-dibaryon rescattering. Crossed contributions and those with the PV interaction instead on the bottom right not displayed.

3.4 No PV Three-Nucleon Interaction at Next-To-Leading Order

Finally, we test whether the spin-isospin-cluster structure of these diagrams matches that of the PV 3NIs, Eq. 3.21. All contain the same PV tree-level diagrams of Fig. 3. Using the PC and PV Lagrangeans (2.1/2.15) and the partial-wave projectors (3.17) onto an incoming $^2\mathrm{S}_{\frac{1}{2}}$- and outgoing $^2\mathrm{P}_{\frac{1}{2}}$-wave, the corresponding structure is:

$$i\mathcal{M}\left[^2\mathrm{S}_{\frac{1}{2}} \to {}^2\mathrm{P}_{\frac{1}{2}}, p, q\right]_{\mathrm{2NI}} = A_{\mathrm{2NI}}^{(a)} \begin{pmatrix} \mathcal{S}_1 & -\mathcal{T} \\ \mathcal{S}_1 & -\mathcal{T} \end{pmatrix} + A_{\mathrm{2NI}}^{(b)} \begin{pmatrix} \mathcal{S}_1 & \mathcal{S}_1 \\ -\mathcal{T} & -\mathcal{T} \end{pmatrix} . \qquad (3.22)$$

Here, $\mathcal{S}_1 = 3g^{(^3S_1-^1P_1)} + 2\tau_3\, g^{(^3S_1-^3P_1)}$ and $\mathcal{T} = 3g_{(\Delta I=0)}^{(^1S_0-^3P_0)} + 2\tau_3\, g_{(\Delta I=1)}^{(^1S_0-^3P_0)}$ are combinations of the PV 2NI strengths. The overall factors $A_{\mathrm{2NI}}^{(a)}$ and $A_{\mathrm{2NI}}^{(b)}$ are different for the PV 2NI being at the top or bottom vertex of Fig. 3. Like the factor A_{3NI} of Eq. (3.21), their dependence on $E, k, p, q, \gamma_{s/t}$ is irrelevant for the present argument, except for the fact that they may contain potential divergences. Their exact forms will be provided in the above-mentioned upcoming article [28]. Under the transformation to the Wigner-basis (2.9):

$$i\mathcal{M}\left[^2\mathrm{S}_{\frac{1}{2}} \to {}^2\mathrm{P}_{\frac{1}{2}}, p, q\right]_{\mathrm{2NI}}^{\mathrm{Wigner}} = A_{\mathrm{2NI}}^{(a)} \begin{pmatrix} 0 & 0 \\ \mathcal{S}_1 + \mathcal{T} & \mathcal{S}_1 - \mathcal{T} \end{pmatrix} + A_{\mathrm{2NI}}^{(b)} \begin{pmatrix} 0 & \mathcal{S}_1 + \mathcal{T} \\ 0 & \mathcal{S}_1 - \mathcal{T} \end{pmatrix} . \quad (3.23)$$

In other words, the structure of the "tree-level" diagrams does *not* contain the only component of the PV 3NIs, i.e. the $^2S_{\frac{1}{2}}(\lambda = 1)$-$^2P_{\frac{1}{2}}(\lambda = 1)$ component (upper-left entry in the cluster matrix).

As seen in the NLO diagrams of Figs. 7 to 9, the PV 2NI kernel of (3.22) are convoluted with LO PC amplitudes and/or NLO corrections in the strong sector. The cluster-space structure of such graphs is obtained by multiplying the 2×2 matrix (3.23) from the left and right with that of:

(1) the LO half- and full-offshell amplitude of Eqs. (2.6/2.10);

(2) the effective-range insertions $c_{0s/t}$ of Eqs. (2.1/2.12);

(3) the NLO parity-conserving 3NI H_0^{NLO} of Eq. (2.1).

However, we demonstrated in Sec. 2.1 that all these terms are diagonal in the Wigner-basis since Wigner-symmetry is approximatively realised in the strong sector. Off-diagonal elements are suppressed by at least one additional power of Q and hence enter at most at N²LO. Any combination therefore results in a matrix which, at NLO, has a zero in the upper left corner, $(\lambda = 1) \leftrightarrow (\lambda' = 1)$:

$$\begin{pmatrix} 0 & a \\ b & c \end{pmatrix} , \qquad (3.24)$$

with the specific form of the entries a, b, c irrelevant for our argument. In addition, off-diagonal elements of the PC LO amplitude are suppressed by $1/q$, making loop integrals more convergent.

As shown in the preceding Section, though, the expressions for the NLO diagrams with the PV 3NI of Eq. (3.21) (Fig. 6) contain at this order a non-vanishing entry only in the upper left corner at this order. Any possible divergence at NLO therefore does not match the cluster-structure of the PV 3NIs. With the exception of an anomalously large coefficient, no reason exists to promote the PV 3NI to either LO or NLO, and it enters not earlier than at N²LO, in agreement with the simplistic counting.

Consequently, the NLO PV diagrams can at most contain divergences which are already renormalised in the parity-conserving sector as in Fig. 2. This is indeed verified by explicit numerical computations in an upcoming publication [28].

In summary, we have demonstrated that no PV 3NIs enter in the Nd system at NLO.

4 Conclusions

We have shown that no parity-violating three-nucleon interaction enters in the nucleon-deuteron system at leading and next-to-leading order in "pionless" Effective Field Theory. The mass dimensions of PV 3NIs suggest that they enter at order ϵQ (N²LO). However, a PV 3NI can *a priori* be included at lower orders if it is needed as counter-term to absorb

divergences in amplitudes containing PV 2NIs. At LO, $\mathcal{O}(\epsilon Q^{-1})$, the superficial degree of divergence of each contribution to the Nd scattering amplitudes is negative, i.e. all amplitudes converge. At NLO, $\mathcal{O}(\epsilon Q^0)$, we analysed the structure of the interactions in the so-called Wigner-basis to show that the quantum numbers of any divergence that might potentially arise do not match those of the only two PV 3NIs with linear momentum dependence. We used that Wigner symmetry is approximatively realised in the UV limit of the strong sector even when effective ranges are included. Thus, only the five couplings of the LO PV two-nucleon Lagrangean contribute up to and including NLO on the parity-violating side.

Since PV 3NIs are absent at NLO, the uncertainties of a PV calculation with EFT($\not\pi$) in this system are $\lesssim 10\%$ and therefore competitive with those of the most ambitious experiments. Extending the analysis to N^{2}LO is thus not only unnecessary at present; it would also lead to more free parameters since additional PV 2NIs enter, and PV 3NIs are expected to contribute as well.

One might question whether a detailed analysis was indeed necessary to arrive at a result which is also suggested by a simplistic derivative-counting. It must however be stressed that the presence of a 3NI already at LO in the parity-conserving sector can enhance parity-violating 3NI contributions. Given the experimental difficulties of extracting the coupling constants of the LO PV 2NI Lagrangean, PV 3NIs at LO or NLO would have significantly compromised the programme to analyse the strengths of parity-violating two-nucleon interactions. Since reliable, model-independent error-assessments are necessary to interpret the data, investigating the renormalisability of EFT($\not\pi$) at NLO with parity-violating interactions is therefore crucial.

The investigation here was limited to nucleon-deuteron observables, i.e. to the iso-doublet three-nucleon system. The extension to 3N iso-quartet channels is left to the future. They are experimentally realised only as a sub-system in processes involving more than 3 nucleons. in that context, PV 3NIs with $\Delta I = 2, 3$ must also be considered. Including electro-magnetic currents will also be addressed.

Finally, the arguments in this publication are general and prove the assumption by exclusion. It was the particularly transparent divergence structure of the PV and PC amplitudes in EFT($\not\pi$) which allowed for rigorous, analytic arguments without resort to a detailed numerical study of cut-off dependences in observables. As a first application of EFT($\not\pi$) in the three-nucleon sector, we have performed a calculation of the neutron spin-rotation in deuterium whose results will be available soon [28]. Schiavilla et al. [10] explored this process in the "hybrid" formalism, in which EFT($\not\pi$) PV interactions are combined with phenomenological wave functions. In this context, we will also provide numerical confirmation of the results presented here and the construction of partial-wave projectors in the Nd system.

Acknowledgements

References

[1] E. G. Adelberger and W. C. Haxton, Ann. Rev. Nucl. Part. Sci. **35**, 501 (1985).

[2] M. J. Ramsey-Musolf and S. A. Page, Ann. Rev. Nucl. Part. Sci. **56**, 1 (2006) [arXiv:hep-ph/0601127].

[3] S. L. Zhu, C. M. Maekawa, B. R. Holstein, M. J. Ramsey-Musolf and U. van Kolck, Nucl. Phys. A **748**, 435 (2005) [arXiv:nucl-th/0407087].

[4] J. Wasem, *Nuclear Parity Violation and Lattice QCD*, presentation at the INT programme 10-01: "Simulations and Symmetries", 20 May 2010, and private communication.

[5] E. Braaten and H. W. Hammer, Phys. Rept. **428**, 259 (2006) [arXiv:cond-mat/0410417].

[6] L. Platter, Few Body Syst. **46**, 139 (2009) [arXiv:0904.2227 [nucl-th]].

[7] M. J. Savage and R. P. Springer, Nucl. Phys. A **644**, 235 (1998) [Erratum-ibid. A **657**, 457 (1999)] [arXiv:nucl-th/9807014].

[8] L. Girlanda, Phys. Rev. C **77**, 067001 (2008) [arXiv:0804.0772 [nucl-th]].

[9] D. R. Phillips, M. R. Schindler and R. P. Springer, Nucl. Phys. A **822**, 1 (2009) [arXiv:0812.2073 [nucl-th]].

[10] R. Schiavilla, M. Viviani, L. Girlanda, A. Kievsky and L. E. Marcucci, Phys. Rev. C **78**, 014002 (2008) [arXiv:0805.3599 [nucl-th]].

[11] A. C. Phillips and G. Barton, Phys. Lett. B **28**, 378 (1969).

[12] V. Efimov and E. G. Tkachenko, Few-Body Syst. **4**, 71 (1988).

[13] H. W. Grießhammer, Nucl. Phys. A **760**, 110 (2005) [arXiv:nucl-th/0502039].

[14] H. W. Grießhammer, Few Body Syst. **38**, 67 (2006) [arXiv:nucl-th/0511039].

[15] H. W. Grießhammer, Nucl. Phys. A **744**, 192 (2004) [arXiv:nucl-th/0404073].

[16] D. B. Kaplan, Nucl. Phys. B **494**, 471 (1997) [arXiv:nucl-th/9610052].

[17] P. F. Bedaque and H. W. Grießhammer, Nucl. Phys. A **671**, 357 (2000) [arXiv:nucl-th/9907077].

[18] D. R. Phillips, G. Rupak and M. J. Savage, Phys. Lett. B **473**, 209 (2000) [arXiv:nucl-th/9908054].

[19] G. V. Skorniakov and K. A. Ter-Martirosian, Sov. Phys. JETP **4**, 648 (1957).

[20] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series and Products, 5th edition, Academic Press, San Diego, 1994

[21] P. F. Bedaque, H. W. Hammer and U. van Kolck, Nucl. Phys. A **676**, 357 (2000) [arXiv:nucl-th/9906032].

[22] G. Gasaneo and J. H. Macek, J. Phys. B **35**, 2239 (2002).

[23] E. Wigner, Phys. Rev. **51**, 106 (1937).

[24] T. Mehen, I. W. Stewart and M. B. Wise, Phys. Rev. Lett. **83**, 931 (1999) [arXiv:hep-ph/9902370].

[25] P. F. Bedaque, G. Rupak, H. W. Grießhammer and H. W. Hammer, Nucl. Phys. A **714**, 589 (2003) [arXiv:nucl-th/0207034].

[26] H. W. Hammer and T. Mehen, Phys. Lett. B **516**, 353 (2001) [arXiv:nucl-th/0105072].

[27] M. R. Schindler and R. P. Springer, arXiv:0907.5358 [nucl-th].

[28] H. W. Grießhammer, M. R. Schindler and R. P. Springer, in preparation.